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Applications of degenerate/non-degenerate modes coupling in an optical waveguide

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Abstract : Mutual coupling between optical modes is essential in the design of integrated optic devices. In this paper we have described how optical energy couples between modes within and between optical waveguides. It has been implicitly assumed so far in the literature that the modes, once formed are unchanging except perhaps through attenuation due to absorption. In reality, simple mechanisms can lead to significant energy exchange among the various modes of a structure. Coupled mode theory describes this energy exchange and serves as the primary tool for designing optical couplers, switches and filters. We have explored a coupling technique that describes the scalar electromagnetic field of a perturbed waveguide in terms of a superposition of modes of the ideal waveguide. In this paper, various applications of degenerate/non-degenerate modes coupling in an optical waveguides are discussed with examples. We have also applied our degenerate coupled mode theory to analyse the co-directional waveguide coupler. The coupling period of the power is verified by Beam Propagation method.

Keywords : Optical waveguide structure, coupled mode theory approach.

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1. Introduction

Consider two proximate single mode optical waveguides. In each waveguide there are two waves, one propagating in the forward direction and one in the backward direction. Energy transfer, *i.e.* coupling, can occur if the evanescent field from one waveguide extends into the core of the neighboring waveguide. The degree to which two modes exchange energy depends on the design of the coupler and the mode structure of the two waveguides. For example, it seems plausible in the structure that the forward wave in one waveguide (A^+) will primarily couple to the forward wave in the adjacent waveguide (B^+). We have restricted our discussion to planar or rectangular waveguides

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in the whole paper. We have followed the development of the coupled mode theory as described by Yariv [1]. Consider the planar step index waveguide structure, with film index n_f of thickness h, and substrate and cover indices n_s and n_c, respectively [7]. The waveguide can support a finite number of guided modes and an infinite number of radiation modes.

In fact this paper is the proof of our Beam Propagation method results with coupled mode theory approach [2] for coupling period. The agreements of the results are excellent. When two orthogonal modes have identical values of propagation constant β, we must use degenerate perturbation methods for solving the equation of motion. Non-degenerate coupled modes theory applies when two modes with different β^s are coupled. Non-degenerate mode generally display weak coupling. There are many examples of structures where two modes with identical propagation constants are coupled. This leads to degenerate coupling and a different approach is required to solve the problem. The primary equation for mode coupling is given by [1–19]

$$\frac{\partial A_j^-}{\partial z} \exp\left\{j\left(\beta_i z + \omega t\right)\right\} - \frac{\partial A_j^+}{\partial z} \exp\left\{-j\left(\beta_i z - \omega t\right)\right\} + c.c.$$

$$= -\frac{j}{2\omega}\frac{\partial^2}{\partial t^2}\int^{+\infty} P_{pert}(x)\Phi_j(x)dx. \qquad (1)$$

Here A_j is the amplitude for mode j and $\Phi_j(x)$ is the normalized amplitude distribution for mode j and also A^-, A^+, P_{pert} are the backward wave, forward wave and perturbation terms respectively. Others terms has usual meaning as in optical waveguide. The detailed derivation and implications of this equation can be found elsewhere [3,7]. We explore non-degenerate coupling first.

2. Non-degenerate coupling case : coupling due to a dielectric perturbation

We begin with the simple case of a perturbed single mode waveguide and see how a dielectric defect can couple a mode's forward wave to its backward wave. Consider the symmetric slab waveguide shown in Figure 1.

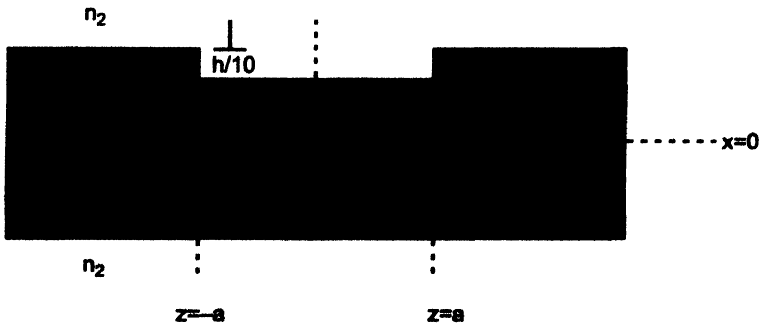

Figure 1. A dielectric waveguide with a notch defect in it acts as a perturbation to the guided modes.

The waveguide consists of a guiding film index, n_1, of thickness h, surrounded by a cladding index, n_2. A small dielectric notch of depth $h/10$ and length $2a$ in the core region perturbs the waveguide structure. Assume that the waveguide is a single mode structure and that for $z > a$ there is no backward traveling wave, $A^{-1}(z > a) = 0$. The mode expects to see an index of n_1 when it gets to the notch region but instead finds the index is n_2. This is a perturbation. To begin our analysis, we must determine the unperturbed modes of the waveguide. We can describe the eigenmode of the symmetric waveguide as [7],

$$\Phi_y = C\exp\left\{-\gamma\left(x-h/2\right)\right\} \quad x \geq h/2$$

$$\Phi_y = C\frac{\cos\kappa x}{\cos\kappa h/2} \text{ or } \frac{\sin\kappa x}{\sin\kappa h/2} \qquad -h/2 \leq x \leq h/2$$

$$\Phi_y = C\exp\left\{\gamma\left(x+h/2\right)\right\} \quad x \leq -h/2. \tag{2}$$

where the coefficient, C is chosen to normalize the power in the waveguide to a value of 1 W per unit length. Here γ refer to the attenuation coefficients in the cover and substrate respectively and κ is the transverse component of wave propagation constant in the guiding film. The specific values of κ, γ and β are determined from the eigenvalue equation [7] for this structure once h, n_1 and n_2 are specified. We assume this has been done and all eigenvalues are known. To determine the coupling, we must describe the perturbation term. In this example, the polarization perturbation, $P_{\text{pert}}(x)$ is the product of the change in dielectric constant $\varepsilon_0\left(n_2^2-n_1^2\right)$ and the electric field of the forward wave $E_1(x)$

$$P_{\text{pert}}(x) = \Delta\varepsilon E_1(x) = \varepsilon_0\left(n_2^2-n_1^2\right)\frac{1}{2}A^+\Phi(x)\exp\left\{-j\left(\beta z-\omega t\right)\right\}+c.c.$$

$$0.4h \leq x \leq 0.5h, -a \leq z \leq a$$

$$= 0 \text{ elsewhere.} \tag{3}$$

Notice that the perturbation term exists only in the notch region. Everywhere else in the waveguide the mode sees an index distribution that it would normally see without the perturbation. The amplitude eq. (1) of motion becomes

$$\frac{\partial A_j^-}{\partial z}\exp\left\{j\left(\beta_i z+\omega t\right)\right\}-\frac{\partial A_j^+}{\partial z}\exp\left\{-j\left(\beta_i z-\omega t\right)\right\}+c.c.$$

$$=-\frac{j}{2\omega}\frac{\partial^2}{\partial t^2}\int_{-\infty}^{+\infty}P_{\text{pert}}(x)\Phi_j(x)dx.$$

$$=-\frac{j}{2\omega}\frac{\partial^2}{\partial t^2}\int_{0.4h}^{0.5h}\varepsilon_0\left(n_2^2-n_1^2\right)|C|^2\left(\frac{\cos\kappa x}{\cos\kappa h/2}\right)^2\left[\frac{1}{2}A^+\exp\{-j(\beta z-\omega t)\}+c.c.\right]dx$$

$$j\omega\varepsilon_0\left(n_2^2-n_1^2\right)\ \frac{1}{2}A^+\exp\left\{-j\left(\beta z-\omega t\right)\right\}+c.c.\Big]\int_{0.4h}^{0.5h}|C|^2\left(\frac{\cos\kappa x}{\cos\kappa h/2}\right)^2 dx$$

$$=j\Gamma\left(A^+\exp\left\{-j\left(\beta z-\omega t\right)\right\}+c.c.\right), \tag{4}$$

where Γ is the coupling constant defined as

$$\Gamma=\frac{\omega\varepsilon_\prime\left(n_2^2-n_1^2\right)}{4}\int_{0.4h}^{0.5h}|C|^2\left(\frac{\cos\kappa x}{\cos\kappa h/2}\right)^2 dx,\ \text{for}\ -a\le z\le a$$

$$=0,\ \text{elsewhere}. \tag{5}$$

Notice that the coupling constant is determined by the overlap of the coupled modes in the perturbation region. In this one-dimensional problem, we only integrate over the x coordinate to determine the coupling coefficient. Multiplying both sides of eq. (4) by $\exp\left\{j\left(\beta z-\omega t\right)\right\}$ yields an equation of motion for the amplitudes

$$\frac{\partial A^-}{\partial z}\cdot\exp\left(2j\beta z\right)-\frac{\partial A^+}{\partial z}+c.c.=j\Gamma A^+. \tag{6}$$

To solve eq. (6), we must integrate both sides. The first term average to a small value when integrated over a distance large compared to $(2\beta)^{-1}$. Thus to first order we can drop the A^{-1} term, leaving the simplified equation of motion

$$\frac{\partial A^+}{\partial z}+c.c.=j\Gamma A^+, \tag{7}$$

which, when integrated over z and applying the initial condition that $A_0=A\left(-a\right)$, has solution

$$A_0=A^+\left(-a\right)\exp\left(-j\Gamma z\right). \tag{8}$$

Thus we see that the amplitude of the field does not change (to first order) but the perturbation (a small region of lower index material) alters the phase of the wave. The forward wave in the region of the perturbation will be (note κ is negative)

$$E(z)={A^+\left(-a\right)}\big/{2}\ \exp\left\{-j\left(\beta+\Gamma\right)z\right\}.$$

$$-a\le z\le a. \tag{9}$$

To find a first order solution for A^-, we must assume that the forward wave, A^+, does not change, so ${\partial A^+}\big/{\partial z}=0$. Except for a slight phase change, this is true. The coupling equation becomes

$$\frac{\partial A^-}{\partial z}\quad j\kappa A^+\exp\left(-2j\beta z\right).\quad -a\le z\le a \tag{10}$$

This can be directly integrated, nothing that

$$A^{-}(a)=0,$$

to yield

$$A^{-}(-a)=-\frac{j\Gamma A^{+}}{\beta}\sin(2\beta a). \tag{11}$$

To first order, the coupling of the forward wave to the backward is maximized when $2\beta a=(q+1/2)\pi$, where q is an integer. In terms of a, the maximum reflections occurs when,

$$a=\frac{(q+1/2)\pi}{2\beta} \tag{12}$$

The behavior of this structure is similar to that of a half wave dielectric stack used to make mirrors. Similarly, choosing a quarter wavelength for a will minimize the reflected term. The maximum amplitude of the reflection depends on Γ/β, which is usually small. Generally speaking, maximum coupling will occur between a forward and backward wave when the spatial frequency of the perturbation is approximately 2β.

2.1. Coupling due to a symmetric notch in a slab waveguide :

Consider the symmetric slab waveguide shown in Figure 3(a). A notch is symmetrically located about $z = 0$. In this waveguide, we will determine the normalized modes of the structure and calculate the coupling between the fundamental *TE* mode and the other *TE* modes of the structure. Assume that $\lambda=1.3\,\mu\text{m}$, $n_1=3.5$ and $n_2=3.498$. The first step is to determine the normalized modes of the ideal waveguide. We find the eigenvalues κ and γ using following equations [7]

$$\tan \kappa h/2=\frac{\gamma}{\kappa} \text{ (For even modes)} \tag{13}$$

$$=-\frac{\kappa}{\gamma}\cdot \text{ (For odd modes)} \tag{14}$$

There are two even modes and two odd modes supported by this structure. Numerically evaluated values for κ and β for each of mode are listed in Table 1.

The electric field distribution for the modes are given by [7]

$$\Phi_y=C\exp\{-\gamma(x-h/2)\} \quad x\geq h/2$$

$$\Phi_y=C\frac{\cos\kappa x}{\cos\kappa h/2} \text{ or } C\frac{\sin\kappa x}{\sin\kappa h/2} \quad -h/2\leq x\leq h/2$$

$$\Phi_y=C\exp\{\gamma(x+h/2)\} \quad x\leq -h/2. \tag{15}$$

We know every variable in these equations except the normalization coefficient, C. To normalize, we must adjust the amplitude, C, to satisfy [7]

$$\int \Phi_n(x)\Phi_m(x)dx = \frac{2\omega\mu}{\beta_i}\delta_i \tag{16}$$

The necessary values of C are listed in Table 1. These mode amplitudes will produce a normalized power of 1 W/cm width in the slab waveguide. Figure 2, shows the calculated mode distributions, for the four normalized modes taken from this data.

Table 1. Propagation coefficients for the four allowed modes.

Mode designation	$\kappa \rightarrow \text{cm}^{-1}$	$\beta \rightarrow \text{cm}^{-1}$	Normalization amplitude (ν/cm)
TE_0	1335.12	169157	99.7
TE_1	2658.1	169142	197.2
TE_2	3949.8	169117	287.7
TE_3	5158.45	169084	353.3

Having found the modes of the ideal waveguide, we can now calculate the coupling constant for each pair of modes. The polarization perturbation in this example is generated by the electric field of the TE_0 mode in the dielectric notches. Explicitly, the perturbation is (for $9\,\mu\text{m} <|x|10\,\mu\text{m}$ and $-a<z<a$)

$$P_{\text{pert}}(x) = \varepsilon_0\left(n_2^2 - n_1^2\right)E_0(x,z,t)$$

$$= -2.647\times10^{-13}\cos(1335.12x)\,\frac{1}{\eta}A_0^+\exp\{-j(\beta z - \omega t)\} + c.c. \tag{17}$$

Knowing P_{pert}, we can next calculate the coupling coefficients between the modes. Plugging the result into eq. (1), the formula for the general coupling coefficient between the TE_{01} and TE_l mode is

$$\Gamma_{0l} =$$

$$\frac{\omega}{2}\left[\int_{0.45h}^{0.5h} -2.647\times10^{-13}\cos(1335.12x)\,\Phi_l + \int_{-0.45h}^{-0.5h} -2.647\times10^{-13}\cos(1335.12x)\,\Phi_l dx\right]. \tag{18}$$

There are two integrals because there are two dielectric regions in the perturbation. By symmetry, we can see that modes TE_1 and TE_3 will not couple to mode TE_0. The product of an even and odd function in a symmetric integral always yields a null result. Therefore two of the coupling coefficients equal zero. The only mode that the TE_0 mode can couple to is the even symmetry TE_2 mode. The coupling coefficient will be

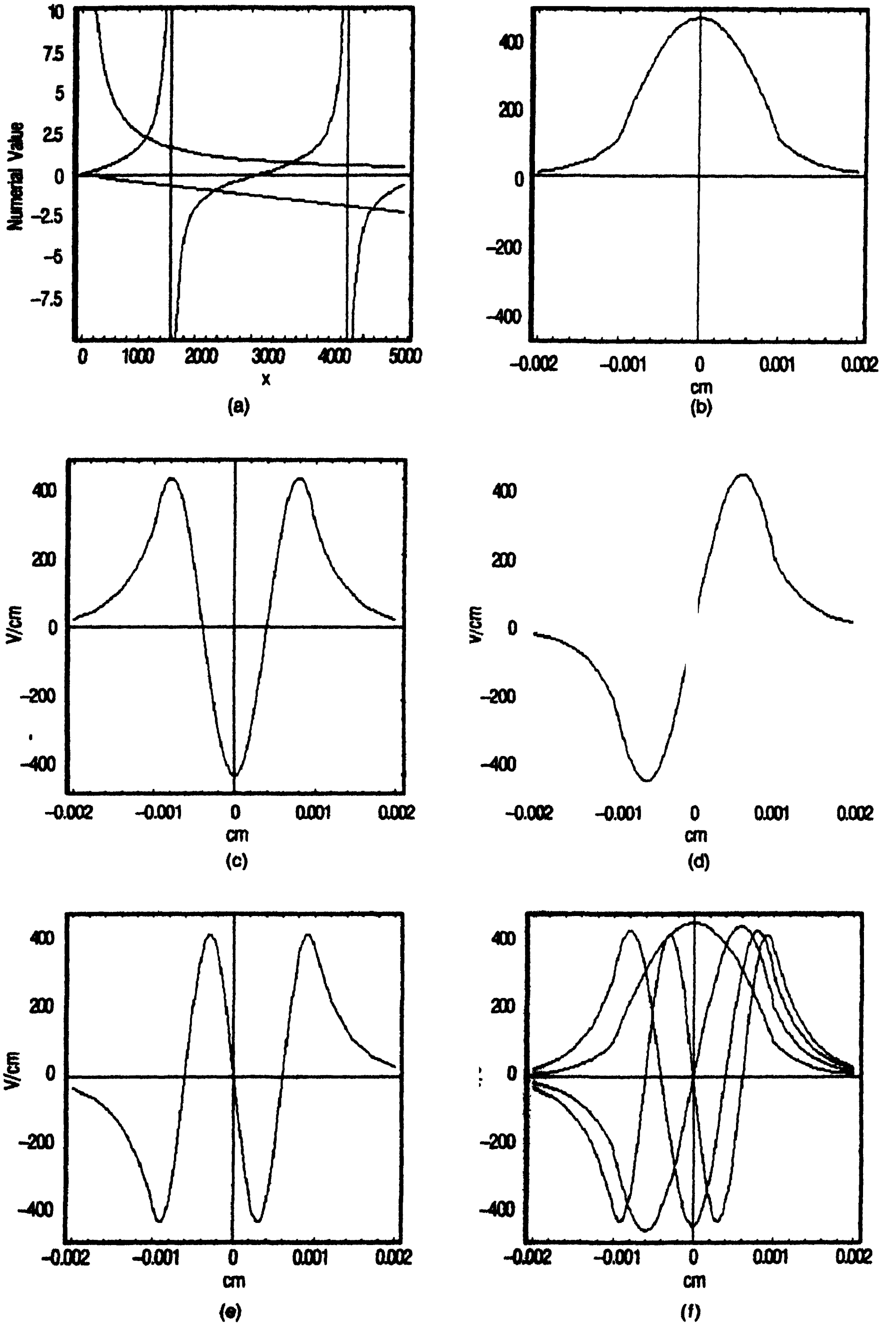

Figure 2. (a) Waveguide in Figure 3(a), supports both even (TE_0, TE_2) and odd (TE_1, TE_3) modes. (b) (c) Even modes. (d) (e) Odd modes. (f) The four mode amplitudes are plotted on the same scale.

$$\Gamma_{02} = 2\frac{\omega}{2}\int_{0.45h}^{0.5h} -2.647\times10^{-13}\cos(1335.12x)\,287.7\,\frac{\cos(3949.8x)}{\cos(3949.8h/2)}\,dx$$

$$= -3.912\,\text{cm}^{-1}. \tag{19}$$

The coupled mode equation relating the TE_2 mode to the TE_0 mode is then,

$$\frac{\partial A_2^-}{\partial z} e^{j(\beta_2 z+\omega t)} - \frac{\partial A_2^+}{\partial z} e^{-j(\beta_2 z-\omega t)} + c.c.$$

$$= j\Gamma_{02} A_0^+ e^{-j(\beta_0 z-\omega t)}, \tag{20}$$

multiplying both sides by $\exp\left\{j\left(\beta_2 z-\omega t\right)\right\}$ converts the equation to

$$\frac{\partial A_2^-}{\partial z} e^{2j\beta_2 z} - \frac{\partial A_2^+}{\partial z} + c.c. = j\Gamma_{02} A_0^+ e^{-j(\beta_0-\beta_2)z}. \tag{21}$$

Integrating over z from $-a$ to a, the first term on the left hand side averages to zero to first order. The amplitude of mode TE_2 at the end of the perturbation, assuming that $A_2^+\left(-a\right)=0$, is

$$A_2^+\left(a\right) = -\left[\frac{2j\Gamma_{02}}{\left(\beta_0-\beta_2\right)}\right] A_0^+ \sin\left(\beta_0-\beta_2\right)a$$

$$= j\left(\frac{7.83}{40}\right) A_0^+ \sin\left(\beta_0-\beta_2\right)a. \tag{22}$$

Squaring this amplitude gives the actual power in the mode. The power in mode TE_2 as a function of perturbation length is shown in Figure 4(a). The coupling length ranges from 0 to 0.1 cm in the plot. We can see that a small fraction (<4%) of the mode energy transfers into mode TE_2 at the optimum interaction length.

3. Degenerate mode coupling

There are many examples of structures where two modes with identical propagation constants are coupled. This leads to degenerate coupling and a different approach is required to solve the problem. Various examples are shown in Figures 3(b), (c). Consider the coupled slab waveguide structure is shown in Figure 3(d). The coupler consists of two identical step index waveguides with core thickness of 10 μm situated approximately 4 μm from each other. To calculate the coupling between the waveguides, we must first determine the eigenmodes of the uncoupled waveguides. Using techniques [7], β was found to be at $\lambda = 1\,\mu\text{m}$, $\beta = 94227\,\text{cm}^{-1}$ (Figure 5(a)) and the normalized mode for the right-hand waveguide is [7]

$$\Phi_y = C\exp\left\{-2840\left(x-0.0012\right)\right\} \quad x \ge 0.0012 \quad \text{cm}$$

$$\Phi_y = C\frac{\cos 1942(x-0.0007)}{\cos 1942\times 0.0005} \qquad 0.0002 \le x \le 0.0012 \quad \text{cm}$$

$$\Phi_y = C \exp\left\{+2840\left(x-0.0002\right)\right\} \qquad x \leq 0.0002 \text{ cm} \tag{23}$$

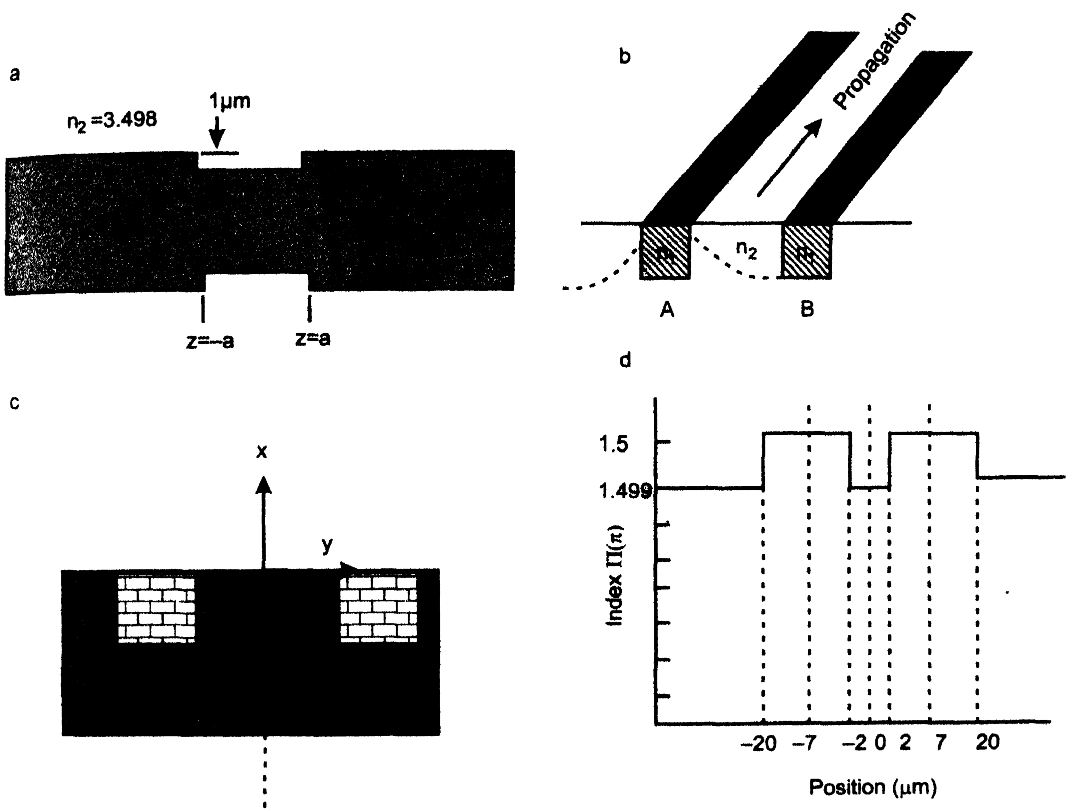

Figure 3. (a) A symmetric waveguide is perturbed by a dielectric notch on the top and bottom of the guiding film. (b) A directional coupler can be made by placing two waveguides in close proximity to one another for a finite distance. (c) The coupled waveguides are separated in the y-direction by a distance of 2a and are 2b wide. (d) Two identical slab waveguides are separated by 4 μm.

The amplitude for the left-hand mode is identical in form, requiring only appropriate offset of the coordinates. The modes are normalized according to eq. (16). Two identical slab waveguides are separated by 4 µm. The eigenmode for one of the slabs is superimposed on the right waveguide, as shown in Figure 5(b). Eq. (16) was numerically evaluated, yielding C = 433.56.

In this example, we assume that initially the mode energy is completely contained in the right hand waveguide. The perturbation therefore only exists in the core of waveguide B, equaling

$$P_{pert}(x) = \Phi(x)\varepsilon_0\left(n_2^2 - n_1^2\right)\left[\frac{1}{2}A^+ e^{-j(\beta z-\omega t)} + c.c. \quad -0.0012 \leq x \leq -0.0002,\right.$$

$$= 0 \text{ Elsewhere} \tag{24}$$

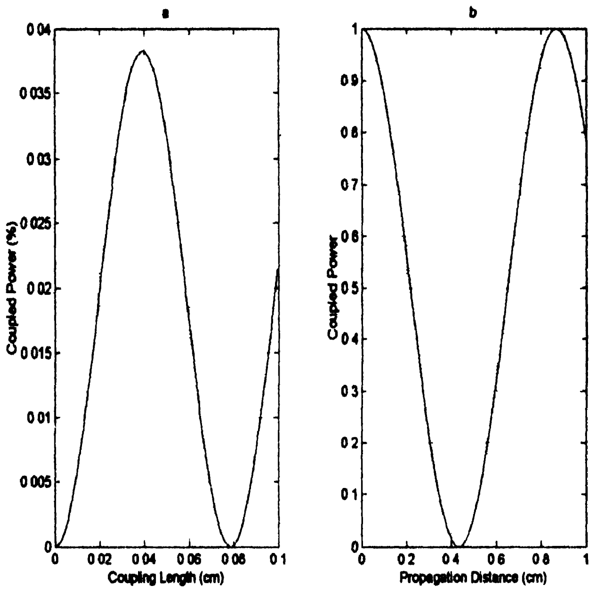

Figure 4. (a) The power transfer from one mode TE_0 to mode TE_2 as a function of notch length in Figure 3(a) The coupling varies periodically with notch length. (b) The power in the original waveguide couples back and forth between the two waveguides. The coupling period for the power is approximately 0.4 cm.

The coupling coefficient is found from

$$\Gamma = \frac{\omega\varepsilon_0}{4}\int_{-0.0012}^{-0.0002}\left(1.5^2 - 1.499^2\right)\Phi_a(x)\Phi_b(x)dx$$

$$= 3.6214\ \mathrm{cm}^{-1} \tag{25}$$

More details data regarding this calculation of coupling coefficient is available with the authors. So how does this result compare to the BPM (Beam propagation method) simulation [19]. Since this is a degenerate coupled system, there will be strong coupling. The amplitude will couple periodically back and forth between the waveguides [8]. The power is proportional to the square of the amplitude. Therefore we would predict the power would couple back and forth as [1–8],

$$|A|^2 = A_0^2 \cos(\Gamma z)^2 . \tag{25}$$

Figure 4(b) shows how the power couples back and forth between the waveguides. By inspection, we see that the spatial coupling period is approximately 0.4 cm, which is exactly what we observed in the BPM work [19].

4. Conclusions

In this paper we have explored some applications of amplitude coupling between two waveguides based on a scalar theory. We found that coupling is maximized when the propagation coefficient, β for the two modes is equal. We can enhance the coupling

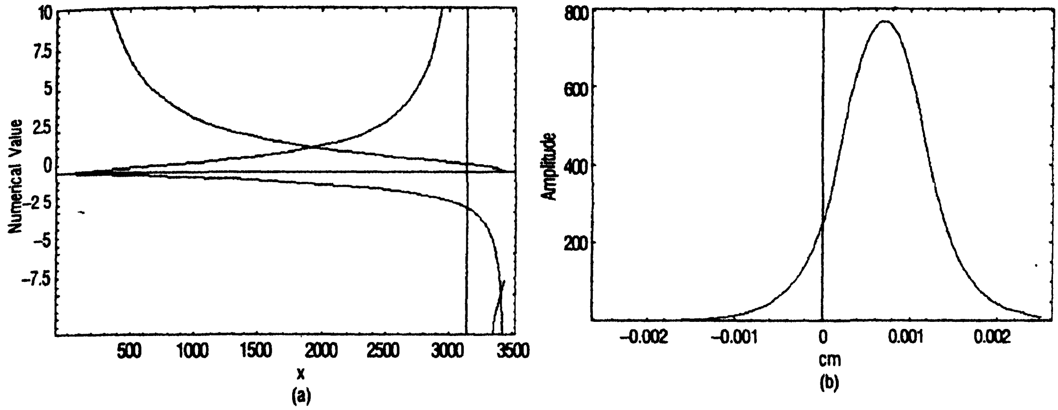

Figure 5. (a) For the waveguide Figure 3(d), there is only one allowed mode, which occurs near κ : 1942.12. (b) The eigenmode for one of the slabs is superimposed on the right waveguide.

between dissimilar waveguides by using phase matching technique. The coupling of radiation from a waveguide into free space is an important problem in optoelectronics. Please note that the more details data are available only with authors of this paper, regarding the detail derivation of coupled mode equations for various cases. Reader should have basic knowledge about integrated optical waveguide to understand this paper. Details of derivation are omitted everywhere to avoid any difficulty to understand the mode coupling phenomena. The calculation is difficult because the guided modes couple to free space modes, which are not normalizable. The design of such couplers is as much an art as it is an engineering task.

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